

# A Parton Model for Diffractive Processes in Deep Inelastic Scattering

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## Abstract

We demonstrate that the global properties of the “rapidity gap” events, observed at HERA, can be understood based on electron-gluon scattering as the underlying partonic process. Using the measured inclusive structure function  $F_2$  to determine the parameters of the parton model, the diffractive structure function  $F_2^D$  is predicted. The ratio of diffractive and inclusive cross sections,  $R_D = \sigma_D/\sigma_{incl} \simeq 1/9$ , is determined by the probability of the produced quark-antiquark pair to evolve into a colour singlet state. This colour singlet cluster may fragment into hadrons independently of the proton remnant, yielding the observed gap in rapidity.

In the “rapidity gap” events, observed in deep inelastic scattering at HERA [1]-[3], the detected hadronic final state has small invariant mass, and it is separated by a gap in rapidity from the proton beam direction. The absence of a hadronic energy flow between proton remnant and current fragment suggests that in the scattering process a colour neutral part of the proton is stripped off which fragments independently of the proton. In analogy to hadronic processes of similar kind the “rapidity gap” events are also called “diffractive” events.

The measured “diffractive” cross section is not suppressed at large values of the momentum transfer  $Q^2$  relative to the inclusive cross section. Such a “leading twist” behaviour is usually regarded as evidence for scattering on point-like constituents. This interpretation, however, appears to be in conflict with the fact that quarks and gluons, the constituents of the proton, carry colour. Hence, one expects the formation of jets in the final state with a hadronic energy flow familiar from ordinary deep inelastic scattering events without a rapidity gap.

In the following we shall demonstrate that this puzzle can be resolved by taking non-perturbative fragmentation effects into account. We start from the production of a quark-antiquark pair in electron-gluon scattering as basic partonic process. Immediately after their production quark and antiquark propagate in the colour field of the proton. With a certain probability, approximated by a statistical weight factor, the quark-antiquark pair evolves into a colour singlet parton cluster which can escape from the proton and fragment independently into hadrons. The cross section for diffractive events can then be calculated in terms of the statistical weight factor, the quark-antiquark production cross section and the gluon density which we determine in terms of the inclusive structure function  $F_2$ . Our approach is related to previous work on “aligned jet models” [4]-[6] and “wee parton lumps” in deep inelastic scattering [7, 8].

Quark-antiquark pairs are produced in electron-gluon scattering. The relevant kinematic variables are (cf. fig. 1)

$$s = (P + k)^2, \quad Q^2 = -q^2 = xys, \quad x = \frac{Q^2}{2P \cdot q}, \quad (1)$$

which characterize inclusive deep inelastic scattering, and the invariant mass  $M$  of the quark-antiquark pair,

$$M^2 = (q + p_g)^2. \quad (2)$$

With  $\vec{p}_g = \xi \vec{P}$  and  $-p_g^2 = m_g^2 \ll Q^2, M^2$  one has,

$$\beta \equiv \frac{Q^2}{Q^2 + M^2} \simeq \frac{x}{\xi}. \quad (3)$$

The differential cross section for the inclusive production of quark-antiquark pairs is given by [9]

$$\frac{d\sigma(ep \rightarrow e(q\bar{q})X)}{dx dQ^2 d\xi} = \frac{\alpha}{\pi x Q^2} g(\xi) \left( \left(1 - y + \frac{y^2}{2}\right) (\sigma_T + \sigma_L) - \frac{y^2}{2} \sigma_L \right) . \quad (4)$$

Here  $g(\xi)$  is the gluon density, and the cross section  $\sigma_{T(L)}$  is obtained by integrating the differential parton cross section over the momentum transfer  $t = (q - l')^2$  (cf. fig. 1) from  $t_{min} = -Q^2/\beta$  to  $t_{max} = -m_g^2/\beta$ ,

$$\sigma_{T(L)} = \int_{t_{min}}^{t_{max}} \frac{d\sigma_{T(L)}}{dt} . \quad (5)$$

The differential parton cross sections (see fig. 1 and the crossed process) read [9]

$$\begin{aligned} \frac{d\sigma_T}{dt} = & \frac{\pi\alpha\alpha_s}{Q^4} \sum_q e_q^2 \beta^2 \left( 4\beta(1-\beta) + \frac{u}{t} + \frac{t}{u} + \frac{2Q^2}{tu} (t+u+Q^2) \right. \\ & \left. - Q^2 m_g^2 \left( \frac{1}{u^2} + \frac{1}{t^2} \right) \right) , \end{aligned} \quad (6)$$

$$\frac{d\sigma_L}{dt} = \frac{8\pi\alpha\alpha_s}{Q^4} \sum_q e_q^2 \beta^3 (1-\beta) , \quad (7)$$

where the sum extends over all quarks whose mass is small compared to  $Q$ .

From eqs. (4) and (5) one obtains the contribution to the inclusive structure function  $F_2(x, Q^2)$ ,

$$\Delta^{(g)} F_2(x, Q^2) = x \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} g(\xi) \left( (\beta^2 + (1-\beta)^2) \ln \frac{Q^2}{m_g^2 \beta^2} - 2 + 6\beta(1-\beta) \right) . \quad (8)$$

This is the result in the “massive gluon” scheme [9]. The virtuality  $-m_g^2$  of the gluon regularizes the collinear divergence at  $t = 0$ . In the more familiar minimal subtraction scheme one obtains a similar expression, where  $m_g$  is replaced by the subtraction scale  $\mu$ , and the finite part is different. At one-loop order, the complete expression for the structure function  $F_2$  reads,

$$F_2(x, Q^2) = x \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} (S_q(\xi) + S_{\bar{q}}(\xi)) (\delta(1-\beta) + \mathcal{O}(\alpha_s)) + \Delta^{(g)} F_2(x, Q^2) . \quad (9)$$

Here  $S_q$  and  $S_{\bar{q}}$  are quark density and antiquark density, respectively.

At small values of  $\xi$  one has,

$$S_q(\xi) , S_{\bar{q}}(\xi) \ll g(\xi) . \quad (10)$$

For simplicity, we shall neglect in the following the quark contribution to  $F_2$ . In this approximation we shall obtain a parameter free prediction for the diffractive structure function. Note, that  $S_q(\xi)$  is different from  $S_q(\xi, Q^2)$ , the quark density at scale  $Q^2$ . Neglecting  $S_q(\xi)$  in eq. (9) essentially amounts to calculating  $S_q(\xi, Q^2)$  in terms of an “intrinsic” gluon density  $g(\xi)$ , corresponding to a scale  $m_g = \mathcal{O}(1 \text{ GeV}^2)$ . A more complete analysis keeping the contribution from  $S_q(\xi)$  would require information about the longitudinal structure function  $F_L$  in order to predict the diffractive structure function.

For the gluon density at small values of  $\xi$  we use the usual parameterization,

$$g(\xi) = A_g \xi^{-1-\lambda}, \quad (11)$$

where  $A_g$  is a constant. Inserting eq. (11) into eq. (8) we can now evaluate the inclusive structure function  $F_2$ ,

$$F_2(x, Q^2) = \frac{\alpha_s}{2\pi} \sum_q e_q^2 x g(x) \int_x^1 d\beta \beta^\lambda \left( (\beta^2 + (1-\beta)^2) \ln \frac{Q^2}{m_g^2 \beta^2} - 2 + 6\beta(1-\beta) \right). \quad (12)$$

A simple approximation for  $F_2$ , valid at small  $x$ , is obtained by choosing  $x = 0$  as lower limit of integration, and by setting  $\lambda = 0$  in the integrand. This yields ( $x \ll 1$ ),

$$F_2(x, Q^2) \simeq \frac{\alpha_s}{3\pi} \sum_q e_q^2 x g(x) \left( \frac{2}{3} + \ln \frac{Q^2}{m_g^2} \right). \quad (13)$$

This expression provides a good description of the H1 measurement of the structure function [11] for  $\lambda = 0.23$ ,  $m_g = 1.0 \text{ GeV}$  and  $A_g \alpha_s \sum_q e_q^2 = 0.61$ . For these parameters (13) is very close to the phenomenological fit of the structure function  $F_2$  given in [11]. We have also evaluated the expression (12) without any approximation. Instead of the simple result (13) one then obtains some cumbersome function of  $x$  and  $\lambda$ . Comparison with the H1 data yields again  $\lambda = 0.23$ , whereas the value of  $m_g$  is now  $0.8 \text{ GeV}$ . The following discussion will be based on the simple analytical expression (13).

We are now ready to calculate the diffractive structure function. The main idea is that the quark-antiquark pair, originally produced in a colour octet state, changes its colour randomly through further soft interactions with the colour field of the proton remnant. Hence, the quark-antiquark pair evolves into a parton cluster which separates from the proton remnant with some probability  $P_8$  in a colour octet state, and with probability  $P_1 = 1 - P_8$  in a colour singlet state. In the first case, there is a colour flow between proton remnant and current fragment leading to the typical hadronic final state. In the latter case, however, the colour singlet final state may fragment independently of the proton remnant, yielding a gap in rapidity. For a sufficiently fast rotation of the colour

spin of quark and antiquark, the probabilities should simply be given by the statistical weight factor accounting for the possible states of the quark-antiquark pair, i.e.,

$$P_1 \simeq \frac{1}{9} , \quad P_8 \simeq \frac{8}{9} . \quad (14)$$

Similar ideas concerning the rotation of quarks in colour space have been discussed by Nachtmann and Reiter [10] in connection with QCD-vacuum effects on hadron-hadron scattering.

In analogy with the usual inclusive structure functions, the diffractive structure functions are defined as,

$$\frac{d\sigma_D}{dx dQ^2 d\xi} = \frac{4\pi\alpha^2}{xQ^4} \left( \left( 1 - y + \frac{y^2}{2} \right) F_2^D(x, Q^2, \xi) - \frac{y^2}{2} F_L^D(x, Q^2, \xi) \right) . \quad (15)$$

The structure function  $F_2^D$  is easily obtained from eq. (8). With  $x = \beta\xi$ , and including the statistical weight factor (14), one obtains

$$F_2^D(x, Q^2, \xi) \simeq \frac{1}{9} \frac{\alpha_s}{2\pi} \sum_q e_q^2 g(\xi) \bar{F}_2^D(\beta, Q^2) , \quad (16)$$

where

$$\bar{F}_2^D(\beta, Q^2) = \beta \left( (\beta^2 + (1 - \beta)^2) \ln \frac{Q^2}{m_g^2 \beta^2} - 2 + 6\beta(1 - \beta) \right) . \quad (17)$$

Since the gluon density  $g(\xi)$  and the mass scale  $m_g$  have been determined by the fit to the inclusive structure function  $F_2$ , the diffractive structure function is unambiguously predicted, including its normalization.

Let us discuss the main properties of this result. An immediate consequence is the prediction of the ratio of diffractive and inclusive cross sections. From eqs. (8)-(10) and (16) one obtains,

$$R_D = \frac{\int_x^1 d\xi F_2^D(x, Q^2, \xi)}{F_2(x, Q^2)} \simeq \frac{1}{9} . \quad (18)$$

Note, that within the model described above, this ratio directly measures the probability of forming a colour singlet parton cluster in the scattering process.

The form of the diffractive structure function (16) is identical with expressions obtained based on the idea of a “pomeron structure function” [12]-[17]. The interpretation of the ingredients, however, is rather different. The “pomeron flux factor” is replaced by the density of gluons inside the proton, which factorizes. The “pomeron structure function” for partons with momentum fraction  $\beta$  inside the “pomeron” is identified as the differential distribution for the production of a quark-antiquark pair with invariant mass  $M^2 = Q^2(1 - \beta)/\beta$ .

The function  $\bar{F}_2^D(\beta, Q^2)$  is plotted in fig. 2 for three different values of  $Q^2$ . For intermediate values of  $\beta$  between 0.2 and 0.6 this function is rather flat. Approximating  $\bar{F}_2^D(\beta, Q^2)$  in this interval by  $\bar{F}_2^D(0.4, Q^2)$ , a comparison of eqs. (9) and (16) yields the scaling relation [8]

$$F_2^D(x, Q^2, \xi) \simeq \frac{D}{\xi} F_2(\xi, Q^2) , \quad (19)$$

where  $D \simeq 0.04$ , independent of  $Q^2$ . Note, that the  $\beta$ -spectrum shown in fig. 2 is rather sensitive to the infrared cutoff  $m_g$  at small and large values of  $\beta$ .

The scaling relation (19) provides a rather accurate description of recent measurements of the diffractive structure function by the H1 collaboration. The experimental data are consistent with

$$F_2^D(x, Q^2, \xi) \propto \ln(Q^2) \xi^{-n} , \quad (20)$$

where  $n = 1.19 \pm 0.06 \pm 0.07$  [3]. This is in good agreement with our results eqs. (16),(17) with  $\lambda = 0.23$ .

Let us finally verify that the model described above predicts indeed the appearance of rapidity gaps. For simplicity, we shall consider the rapidity of the antiquark with momentum  $l$  in the  $\gamma^*p$ -rest frame (cf. fig. 1). It is related to other kinematical variables by

$$\eta = \frac{1}{2} \ln \left[ \xi(1 - \beta) \frac{u + m_g^2 \beta}{t + m_g^2 \beta} \right] . \quad (21)$$

From eq. (4) one obtains,

$$\begin{aligned} \frac{d\sigma_D}{dydQ^2d\xi d\eta} &= \frac{dt}{d\eta} \frac{d\sigma_D}{dydQ^2d\xi dt} \\ &= \frac{dt}{d\eta} \frac{1}{9} \frac{\alpha}{\pi y Q^2} g(\xi) \left( \left( 1 - y - \frac{y^2}{2} \right) \left( \frac{d\sigma_T}{dt} + \frac{d\sigma_L}{dt} \right) - \frac{y^2}{2} \frac{d\sigma_L}{dt} \right) . \end{aligned} \quad (22)$$

The total diffractive cross section for a maximum rapidity  $\eta_{max}$  can now be obtained by integrating over a specified kinematic domain, where the rapidity of the antiquark is larger than the rapidity of the quark. The reverse configurations yield the same contribution, resulting in a factor of 2. Hence, one obtains

$$\frac{d\sigma_D}{d\eta_{max}} = 2 \int_{Q_1^2}^{Q_2^2} dQ^2 \int_{y_1}^{y_2} dy \int_x^{\xi_{max}} d\xi \frac{d\sigma_D}{dydQ^2d\xi d\eta} , \quad (23)$$

where  $\xi_{max} = \min\{1, x + \exp 2\eta_{max}\}$ . Using the kinematic boundaries  $0.03 < y < 0.7$  and  $7.5 \text{ GeV}^2 < Q^2 < 70 \text{ GeV}^2$  [3] one obtains the distribution shown in fig. 3. Note, that the approximation  $m_g^2 \ll Q^2$  has been used throughout the calculation. Above  $\eta_{max} \sim 2$  the diffractive cross section is clearly negligible. Assuming that the final state hadrons

are produced in the rapidity interval spanned by the quark-antiquark pair, one obtains the observed rapidity gap.

In summary, we have demonstrated that electron-gluon scattering can account for the global properties of the “rapidity gap” events observed at HERA provided the following two hypotheses concerning the formation of the final state are correct: First, the initially produced quark-antiquark pair evolves with a probability, given by a statistical weight factor, into a colour singlet parton cluster which fragments independently of the proton remnant; second, the rapidity range of the diffractive hadronic final state is essentially given by the rapidity interval spanned by the produced quark-antiquark pair.

This simple picture appears to provide a rather accurate description of the observed diffractive events, including the total rate, the  $\xi$ -dependence and the  $Q^2$ -dependence. This agreement with experimental data may appear fortuitous, since a number of theoretical issues still remain to be settled. These include the non-perturbative formation of the colour singlet cluster and the role of the infrared cutoff. Our results indicate, that an appropriate starting point for the evaluation of the inclusive as well as the diffractive structure functions at small values of  $x$  may be a semiclassical approach where “wee partons” are treated as a classical colour field. We expect that such a framework will lead to results very similar to the ones described above.

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## Figure captions

**Fig.1** Quark-antiquark pair production in electron-gluon scattering.

**Fig.2** Dependence of the diffractive structure function on  $\beta$  and  $Q^2$ .

**Fig.3** Distribution of the maximal rapidity in the  $\gamma^*p$ -rest frame defined by the most forward quark or antiquark.



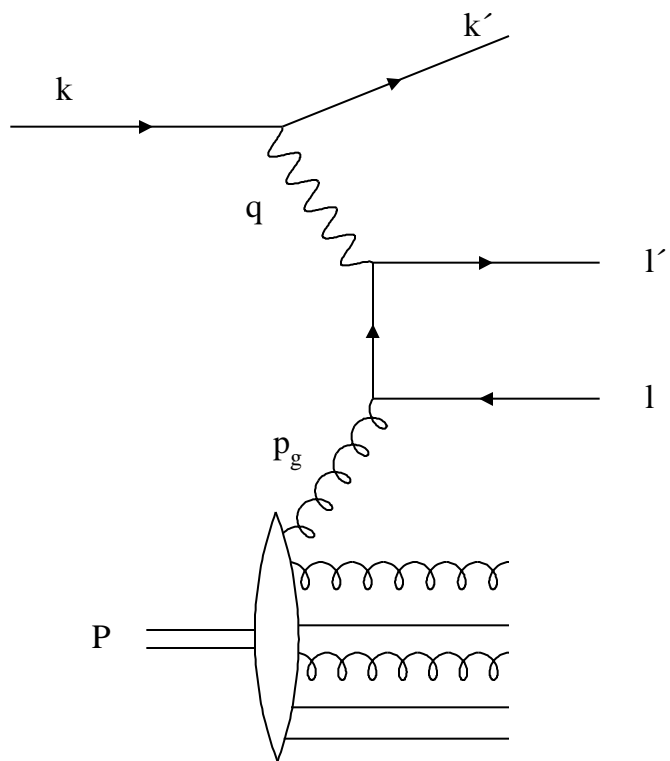


Fig. 1

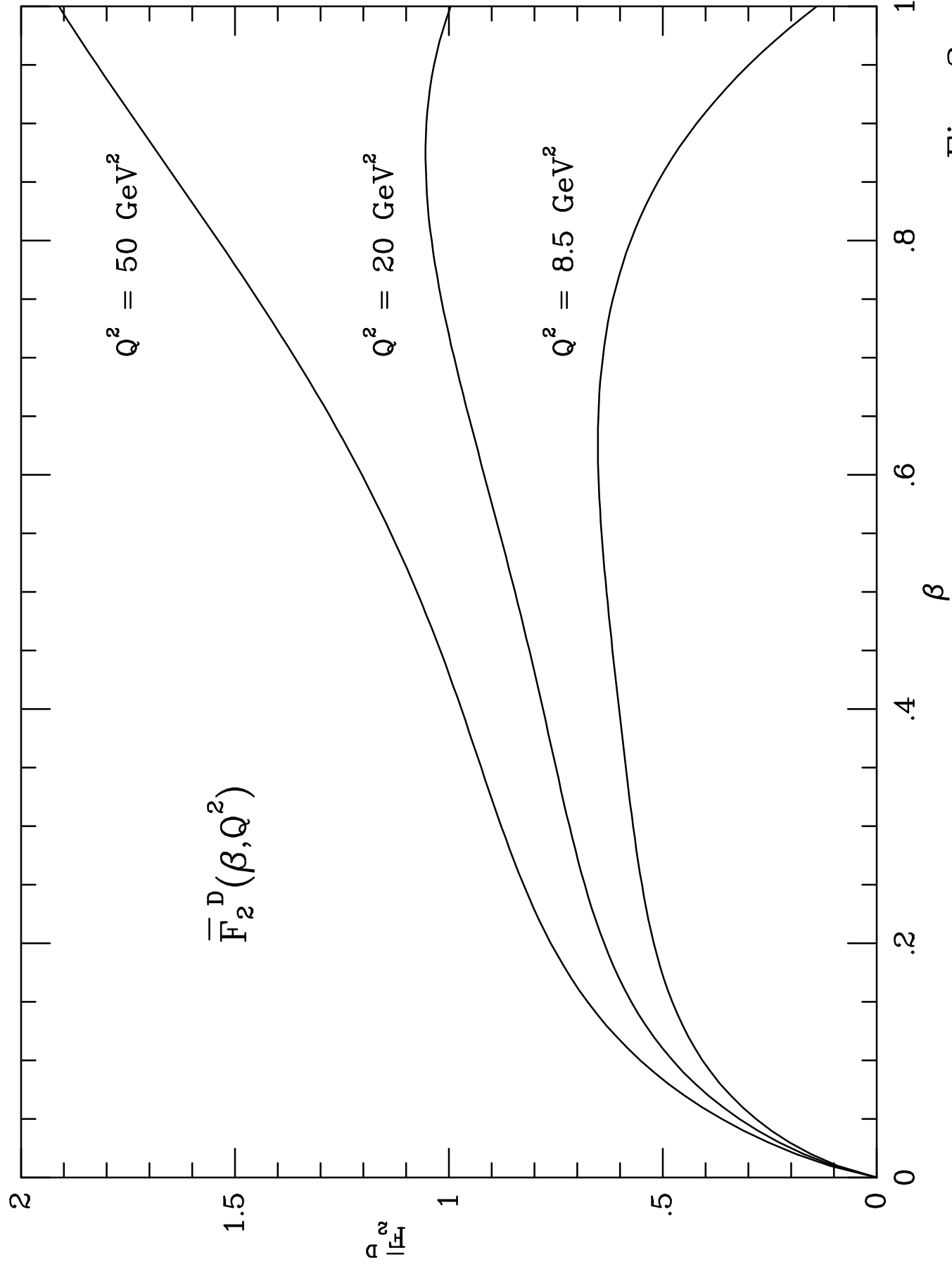


Fig. 2

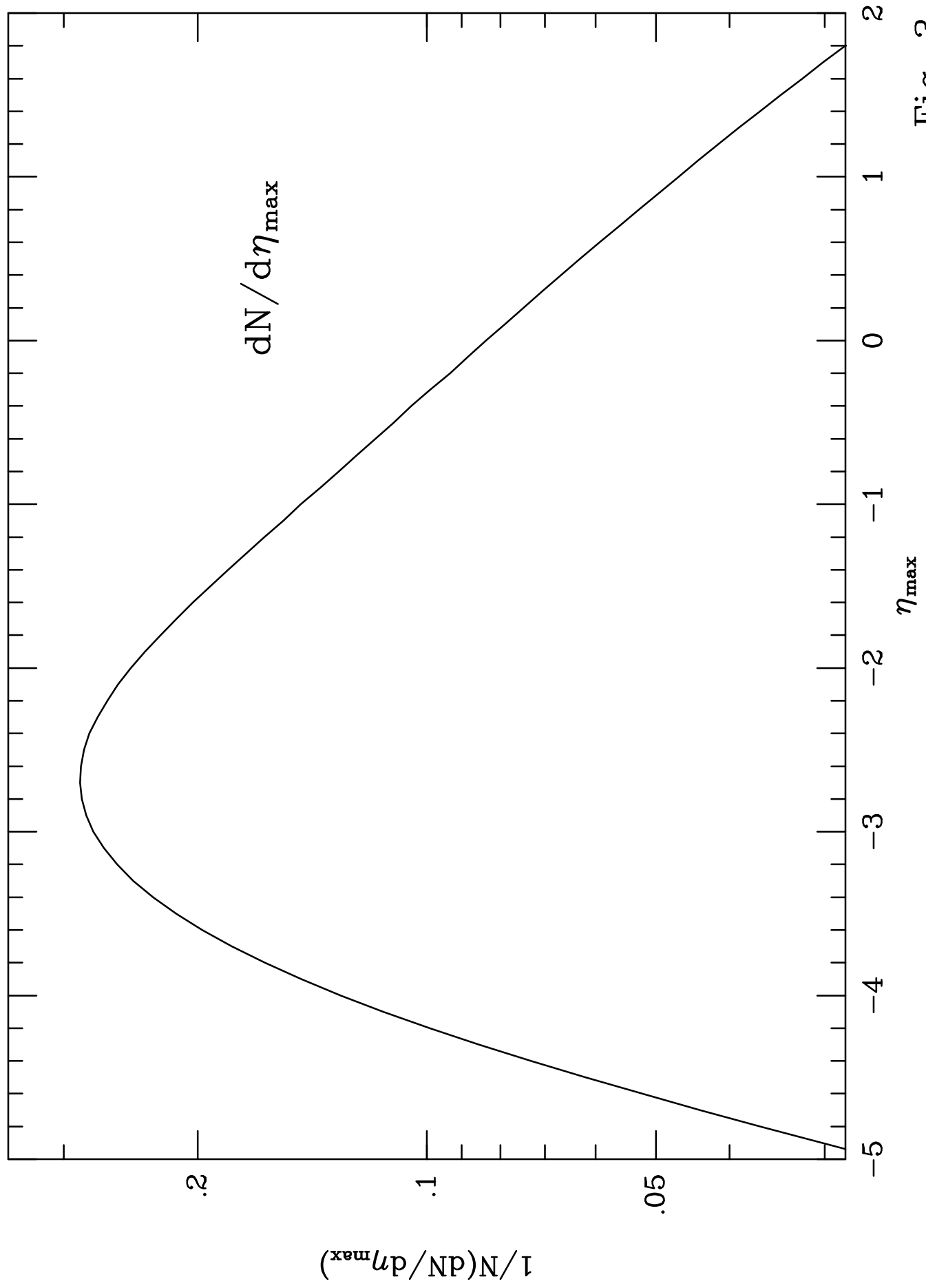


Fig. 3